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Modeling the Deposition of Colloidal Particles on Solid Spheres

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ABSTRACT

A theory describing the deposition of monodisperse and polydisperse carbon particles onto the surface of monodisperse and polydisperse plastic spheres in simple shear flow was developed. The polydispersity of the plastic and the carbon particles was described by log normal distributions. Smoluchowski's equation for the collision frequency between the carbon and the plastic spheres was used. The theory involves few assumptions, the most important one being that the carbon particle occupies an area on the plastic surface equal to its projected area. Expressions were derived to calculate the orthokinetic deposition efficiency and hence the deposition rate at the early stages of the deposition phenomenon. Comparing theoretically calculated with experimentally measured values of the number of carbon particles enables calculation of the orthokinetic deposition efficiency.

INTRODUCTION

Adsorption of gas molecules on a surface of a suitable adsorbent has been thoroughly investigated (1). Adsorption of polymer molecules onto various types of surfaces under different flow conditions has been studied by van de Ven et al. (2, 3). They also examined the deposition of colloidal particles on a glass surface both theoretically and experimentally using the impinging jet technique (4, 5). Polverari and van de Ven (6) studied the electrostatic and steric interactions in particle deposition by the evanescent wave light technique. Deposition of filler particles on pulp fibers was investigated by van de Ven et al. (7–9). Shear-induced deposition of colloidal particles on spheroids was theoretically studied by Petlicki and van de Ven (10).

Mousa and Qasaymeh (11) experimentally investigated the removal of ink particles from recycled newspaper using plastic spheres. They found that ink particles deposit on the surface of plastic spheres, and that the latter can be easily removed to yield clean pulp fibers. In this paper a theoretical investigation of the deposition of carbon particles on the surface of plastic spheres in simple shear flow is presented. The collision frequency between the carbon and the plastic spheres was expressed using Smoluchowski's equation (12–14). Experimental investigation of the theory developed in this paper will be presented in a future paper. We should point out that the analysis shown in this paper is general and can be applied to any kind of spherical solid particle. The theory can also be applied to nondeformable emulsion droplets.

THEORETICAL ANALYSIS

Consider a unit volume of a suspension containing both plastic and carbon particles subjected to a simple shear. Due to this shearing action the carbon particles will collide with the plastic spheres. Some of these collisions will be successful and lead to deposition of the carbon particles on the surface of the plastic spheres. Hence the number of carbon particles per unit volume in the suspension will decrease with time. In order to derive a model describing the change in the number of carbon particles per unit volume due to the deposition process, the following assumptions will be made. First, the bonding between the plastic and the carbon particles is strong enough that no carbon particle will detach from the surface of the plastic spheres. This assumption is valid for the removal of ink particles from a wastepaper pulp suspension by plastic particle where special surface-active agents are added to increase the affinity of ink to the plastic. Second, the carbon–carbon particle coagulation rate is very weak compared to the carbon–plastic deposition rate. This assumption was tested by conducting a preliminary test on the stability of carbon suspension against shear. The test involved counting the number of carbon particles and measuring the intensity of the transmitted light through the carbon suspension. The results showed that both measurements remained unchanged. Hence, the only mechanism for the loss of carbon particles is by their deposition on the surface of the plastic spheres. This assumption also indicates that a single layer of carbon particles (monolayer coverage) exists around the plastic spheres. The third assumption is that the analysis is valid for low area coverage. In this case both the large and the small carbon particles have equal access to the surface of the plastic spheres.

Finally the orthokinetic deposition efficiency is independent of the radii of the carbon and the plastic spheres. A mass balance on this unit volume leads to

$$dn/dt = -\upsilon\alpha(1 - \theta) \tag{1}$$

where n is the number of carbon particles per unit volume, α is the orthokinetic deposition efficiency which is a measure of the fraction of successful collisions that leads to deposition, θ is the fractional area coverage, i.e., it is the ratio of the area of the plastic spheres covered with carbon particles to its total surface area, and υ is the collision frequency between carbon and plastic spheres per unit volume per unit time. The following three cases will be considered: 1) Both plastic and carbon particles are monodisperse in size. 2) The plastic spheres are monodisperse whereas the carbon particles are polydisperse. 3) Both the plastic and the carbon particles are polydisperse in size.

Monodisperse Carbon Particles

The collision frequency can be expressed using Smoluchowski's equation written as (12–14)

$$\upsilon = \frac{4}{3} GnN(a + A)^3 \tag{2}$$

where G is the shear rate, a is the radius of the carbon particles, and A and N are the radius and number per unit volume of plastic spheres, respectively. The fractional area coverage, θ, can be expressed in terms of the number and size of both plastic and carbon particles as follows: Let n_0 be the number of carbon particles per unit volume at time 0, and let n_c be the number of carbon particles per unit volume deposited on the surface of plastic spheres. Then at any time t we can write

$$n_0 = n_c + n \tag{3}$$

Now each carbon particle when deposited on the surface of plastic particle occupies a surface area equals to its projected area, which is equal to πa^2. Therefore we can write

$$\theta = \frac{\text{area occupied by the carbon particles, } A_c}{\text{total surface area of the plastic particles, } A_p} = \frac{a^2 n_c}{4A^2 N} \tag{4}$$

Substituting Eq. (3) into Eq. (4):

$$\theta = \frac{a^2 n_0(1 - n')}{4A^2 N} = C(1 - n') \tag{5}$$

where $n' = n/n_0$ and $C = a^2n_0/4A^2N$. Upon substituting Eqs. (2) and (5) into Eq. (1):

$$\frac{dn}{dt} = -\frac{4}{3}GnN(a + A)^3\alpha[1 - C(1 - n')] \tag{6}$$

In dimensionless form, the above equation can be rewritten as

$$dn'/dt' = -n'[1 - C + Cn'] \tag{7}$$

where $t' = 4/3\ GN(A + a)^3\alpha t$. The solution of the above equation is

$$n' = \frac{1 - C}{\exp[(1 - C)t'] - C}, \quad C \neq 1 \tag{8a}$$

$$= \frac{1}{t + 1}, \quad C = 1 \tag{8b}$$

Note that C represents the total projected area of the carbon particles present in the suspension initially to the total surface area of the plastic spheres.

Polydisperse Carbon Particles

Let the carbon particles be polydisperse in size and be described by a log normal distribution of a mean size a_m and a standard deviation σ. If this distribution is divided into classes such that the mean size and the number of particles of each class are a_i and n_i, respectively, then the collision frequency between the particles in each class and the plastic spheres can be rewritten as (12–14)

$$\upsilon = \frac{4}{3}GN\sum_i n_i(a_i + A)^3 \tag{9}$$

In this case the surface area coverage can be expressed as

$$\theta = \frac{\sum_i a_i^2(n_{0,i} - n_i)}{4A^2N} = Cl\sum_i a_i'^2(n_{0,i}' - n_i') \tag{10}$$

where $n_{0,i}$ is the number of carbon particle in class i at $t = 0$, $n_{0,i}' = n_{0,i}/n_0$, $n_i' = n_i/n_0$, $a_i' = a_i/A$, and $Cl = n_0/4N$. Hence Eqn. (9) for a polydisperse carbon dispersion becomes

$$\frac{d\sum_i n_i}{dt} = \frac{dn}{dt} = -\frac{4}{3}\alpha NGA^3\sum_i n_i(1 + a_i')^3\left[1 - Cl\sum_i a_i'^2(n_{0,i}' - n_i')\right] \tag{11}$$

In dimensionless form, Eq. (11) becomes

$$\frac{d \sum_i n_i'}{dt'} = \frac{dn'}{dt'} = \sum_i n_i'(1 + a_i')^3 \left[1 - C1 \sum_i a_i'^2(n_{0,i}' - n_i')\right] \quad (12)$$

where t' is now defined as $t' = 4/3\ G\alpha NA^3t$ and $n' = \sum_i n_i'$.

Polydisperse Carbon and Plastic Spheres

If the plastic spheres are also polydisperse in size, that can be represented by a log mean distribution of mean size A_{m} and a standard deviation σ_{p}. Then for this situation, Eq. (12) becomes

$$\frac{d \sum_i n_i'}{dt'} = \frac{dn'}{dt'} = \left[\sum_i \sum_j n_iN_j(a_i + A_j)^3\right]\left[1 - \frac{C2 \sum_k (n_{0,k}' - n_k')a_k^2}{\sum_k N_{0,k}'A'^2}\right] \quad (13)$$

where $C2 = n_0/4N$, $N_{0,k}' = N_{0,k}/N$, and $t' = 4/3\ G\alpha Nt$. Equations (12) and (13) were solved numerically using the Runga–Kutta fourth-order integration technique.

RESULTS AND DISCUSSION

The change in the number of monodisperse carbon particles and hence the deposition rate as a function of the dimensionless time, t', for various values of C can be seen in Fig. 1. Since the mathematical derivation is valid for a small fractional area coverage, a small dimensionless time scale was used. As expected, the number of carbon particles decreases as the shearing time increases. Moreover, the change in the number of particles is faster at the beginning and then it slows down. This is because the chance for more particles to deposit on the surface of the plastic spheres becomes smaller as time proceeds since the surface area available for deposition becomes smaller. It can be noticed that as the value of C increases, the deposition rate becomes smaller. This is not surprising if we notice that the value of C represents the ratio between the total projected area of the carbon particles to the actual surface area of the plastic spheres that the carbon particles can occupy. One can also say that the value of C is proportional to the ratio of the amount of carbon to the amount of plastic spheres existing in the suspension. Hence, a high value of C means that there is more than enough carbon to deposit on the surface of the plastic spheres. Therefore $n' = n(t')/n_0 = (n_0 - n_c')/n_0$ is slightly affected

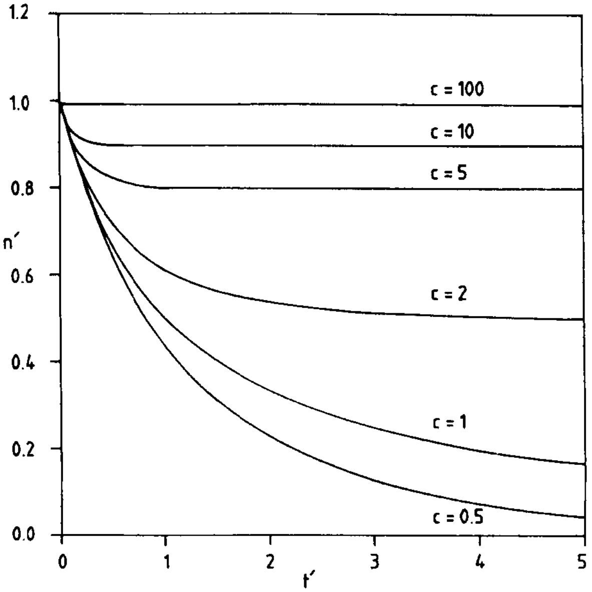

FIG. 1 Change in n' versus t' for monodisperse carbon particles of radius 2.5 μm for various values of C.

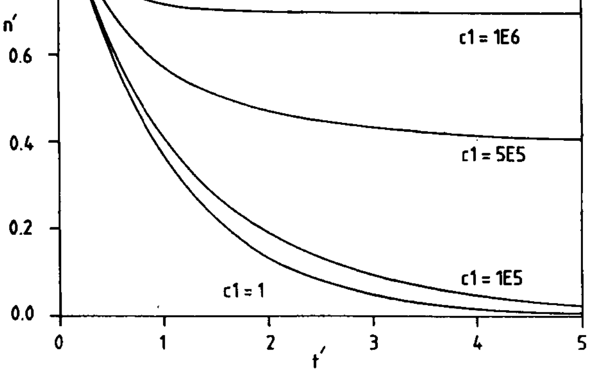

FIG. 2 Change of n' versus t' for various values of $C1$ for polydisperse carbon particles. The distribution is log normal with a_m = 2.5 μm and σ = 0.3. The plastic spheres are monodisperse with A = 1500 μm.

since $n_c \ll n_0$. But when the value of C is small, n' is affected much more rapidly since n_c is comparable to n_0 and hence n' changes faster. Therefore, the calculations were discontinued at early t' when the values of C were large in order to guarantee that the area coverge is low.

The change in n' versus t' for polydisperse carbon particles is shown in Fig. 2. The carbon particles are considered log normally distributed with a mean radius of 2.5 μm and a standard deviation of 0.3 μm. The effect of the value of the parameter $C1$ can also be seen in Fig. 2. Again as expected, n' decreases as the shearing time increases. Also, as the value of $C1$ increases, n' increases. The validity of the calculations can be tested for a value of $C1 \rightarrow \infty$. In this case the amount of plastic spheres $\rightarrow 0$ since $C1 = n_0/4N$. It is expected for this situation that no deposition should take place and hence no change in n' should be seen, as the calculations in Fig. 2 indicate. Again, since the above derivation is valid for low area coverage, the calculations were ceased at early t' values in order to have high values of $C1$. The change in the particle size distribution for different values of t' is portrayed in Fig. 3.

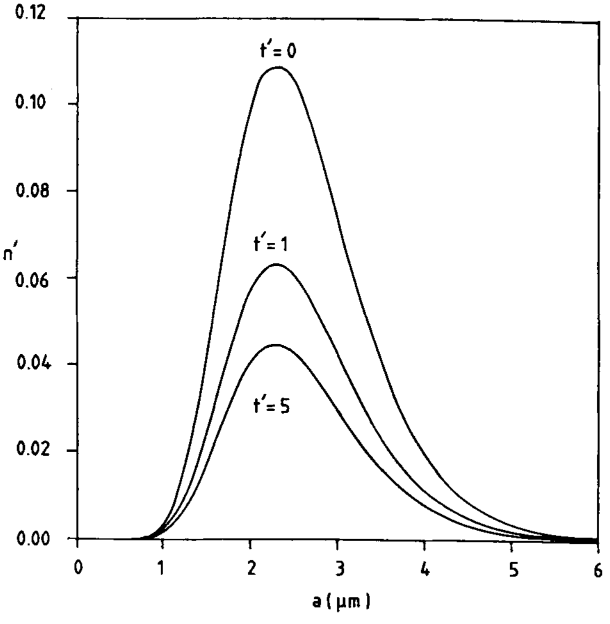

FIG. 3 Change in the carbon particle size distribution due to their deposition on the surface of plastic spheres for $C1 = 500{,}000$ and various values of t'. The carbon particles are log normally distributed with $a_m = 2.5$ μm and $\sigma = 0.3$. The plastic spheres are monodisperse with $A = 1500$ μm.

The effect of the mean particle radius on the change of n' with t' is depicted in Fig. 4. It can be observed that as a_m increases, the deposition rate becomes smaller. The reason for this is that the projected area of the carbon particle is proportional to the square of its radius. Hence, few particles are required to cover the surface of a plastic sphere. In turn, the change in n' becomes less and hence the deposition rate becomes smaller.

The effect of the polydispersity of the carbon particles expressed in terms of the standard deviation is illustrated in Fig. 5. It can be seen that as the standard deviation increases, the rate of deposition decreases. This is because large particles are present in the suspension as the standard deviation increases. These large particles in turn occupy a large surface area on the plastic spheres, and hence the deposition rate decreases.

The effect of plastic spheres' size on the change of n' with t' is shown in Fig. 6. It can be seen that the deposition rate decreases as the plastic spheres' size becomes smaller. This can be attributed to two reasons. First, as the radius of a plastic sphere increases, more surface will be available for the deposition of carbon particles and vice versa. Hence n'

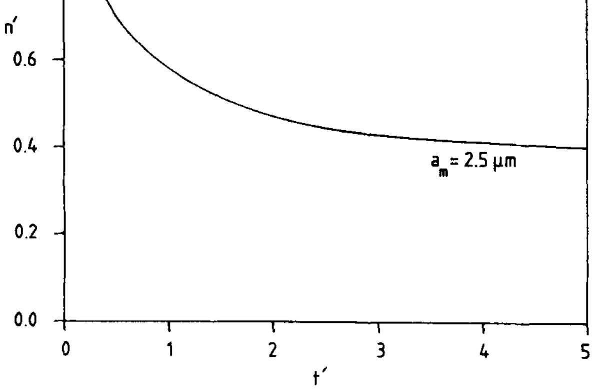

FIG. 4 Change of n' versus t' due to the deposition of log normally distributed carbon particles of $\sigma = 0.3$ and mean radii of 2.5, 5, 7.5, and 10 µm on the surface of monodisperse plastic spheres with a mean radius of 1500 µm.

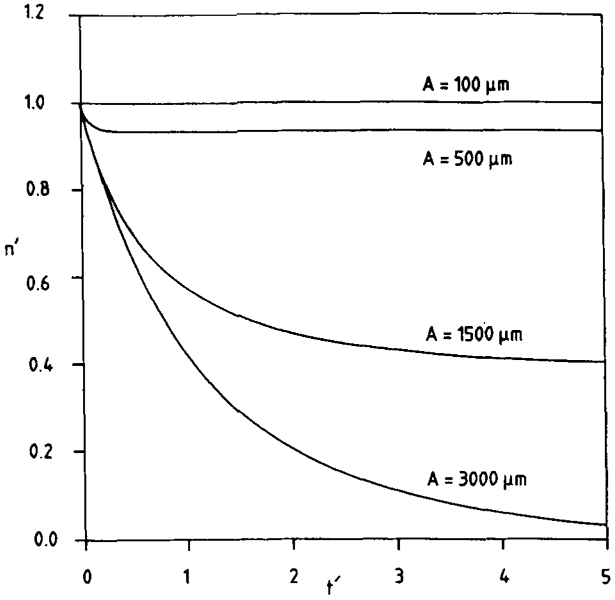

FIG. 5 Change of n' versus t' for log normally distributed carbon particles of a_m = 2.5 μm and σ = 0.1, 0.3, 0.5, and 0.7. The plastic spheres are monodisperse with A = 1500 μm. The value of CI = 500,000.

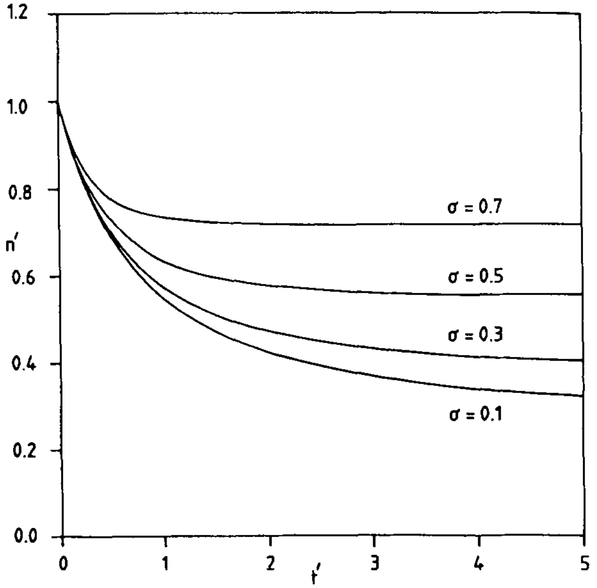

FIG. 6 Change in n' versus t' of carbon particles due to their deposition on the surface of monodisperse plastic spheres with mean radii of 100, 500, 1500, and 3000 μm. The carbon particles are log normally distributed with a_m = 2.5 μm and σ = 0.3.

will decrease faster as t' increases. Second, the capture cross-section of a plastic spheres is proportional to its size (15). Therefore, more particles are captured when the plastic spheres are larger in size.

The change in n' for the case where both the carbon and the plastic spheres are polydisperse is shown in Fig. 7. The results are in accordance with the findings in the previous discussion.

A final point must be added here regarding the change of the deposition rate with time. It can be noticed from all the figures discussed above that the deposition rate is fast initially and then decreases. This is due to the fact that at low shearing time the surface area available for deposition is high, hence the deposition rate is high. But as time proceeds the surface area available for deposition gradually diminishes, and this result in a gradual decrease in the deposition rate.

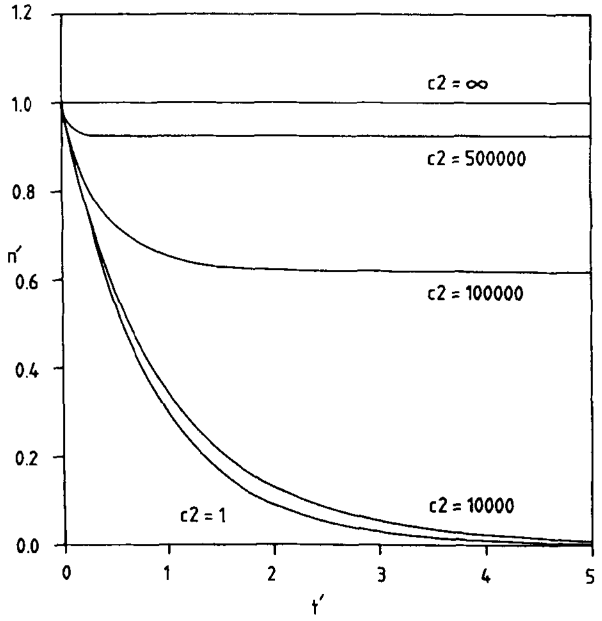

FIG. 7 Change in n' versus t' of carbon particles due to their deposition on the surface of plastic spheres for various values of $C2$. The carbon and the plastic particles are both log normally distributed with $a_m = 2.5$ μm and $\sigma = 0.3$ for the carbon particles and $A = 1500$ μm and $\sigma = 0.2$ for the plastic particles.

Determination of the Orthokinetic Deposition Efficiency, α

The above equations can be useful in determining the orthokinetic deposition efficiency. The change in the number fraction of the carbon particles can be theoretically calculated using either Eq. (8), (12), or (13), depending on whether the carbon and the plastic spheres are mono- or polydisperse. Further, the change in n' versus the real time can be measured experimentally. Therefore, at any given time t, the value of n' measured experimentally is now known. From the same value of n' calculated theoretically, the value of t' is determined (see Fig. 8). Hence, from the data available about the sizes of the carbon and the plastic spheres (or their size distribution) and the number of particles initially present, the value of α can be estimated. For a mondisperse plastic and polydisperse carbon particles for example: $\alpha = 3t'/4GNA^3t$. The values of t' and t can be read from a figure such as Fig. 8, and the rest of the parameters can be obtained from the initial experimental conditions.

The value of α at $t = 0$ is often more interesting to know since, as we noticed earlier, the deposition rate is fast at low values of t. To estimate α, the theoretical value of dn'/dt' at $t \to 0$ is found from the solution of either Eq. (10), (12), or (13) as presented in Figs. 1–7. Moreover, the value of dn/dt at $t \to 0$ is determined from experimentally measuring n with t. Hence the value of α at $t = 0$ can be determined from the following equations:

$$\alpha = \frac{\left(\frac{dn}{dt}\right)_{\text{exp}}}{\frac{4}{3} G n_0 N (a + A)^3} \quad \text{(for monodisperse carbon particles)} \qquad (14)$$

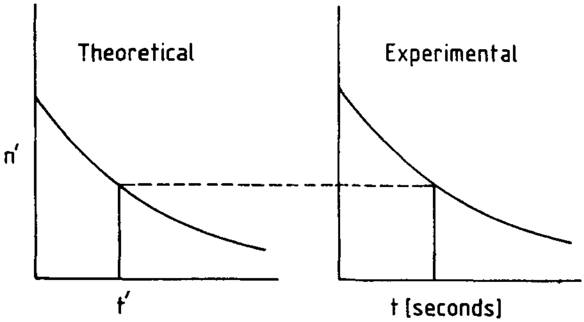

FIG. 8 Illustration of the method used to estimate α. The values of t' and t can be read for a given measured value of n' where α can be estimated by knowing the initial experimental conditions.

$$\alpha = \frac{\left(\dfrac{dn}{dt}\right)_{\text{exp}}}{\dfrac{4}{3} GN \sum_{i} n_i(a_i + A)^3} \quad \text{(for polydisperse carbon particles)} \tag{15}$$

$$\alpha = \frac{\left(\dfrac{dn}{dt}\right)_{\text{exp}}}{\dfrac{4}{3} GN_0 n_0 \sum_{i} \sum_{j} n_i N_j(a_i + A_j)^3} \quad \text{(for polydisperse carbon and plastic spheres)} \tag{16}$$

CONCLUDING REMARKS

Theoretical expressions describing the change in the number of carbon particles subjected to simple shear flow have been derived and solved. The expressions considered the distributions of monodisperse and polydisperse carbon and plastic spheres. The orthokinetic deposition efficiency can be found from experimentally measured n versus t and theoretically calculated n' versus t' values. The initial value of the orthokinetic deposition efficiency may be found using the expressions in Eqs. (14)–(16). Although a carbon suspension was used to study carbon deposition on plastic spheres, the theory can be applied for the removal of ink particles from a wastepaper suspension by plastic spheres.

NOMENCLATURE

a	radius of the carbon particle (m)
A	radius of the plastic particle (m)
C	constant defined in the text (dimensionless)
$C1$	constant defined in the text (dimensionless)
$C2$	constant defined in the text (dimensionless)
G	shear rate (s^{-1})
n	number of carbon particles per unit volume (m^{-3})
N	number of plastic spheres per unit volume (m^{-3})
t	time (s)

Greek

α	orthokinetic deposition efficiency (dimensionless)
θ	fractional area coverage (dimensionless)
υ	collision frequency ($\text{m}^{-3}\cdot\text{s}^{-1}$)
σ	standard deviation (dimensionless)

Subscript and Superscript

0	initially at $t = 0$
c	carbon particles already deposited on the surface of plastic spheres
m	mean
p	plastic
i, j, k	class i, j, or k
	dimensionless parameter

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